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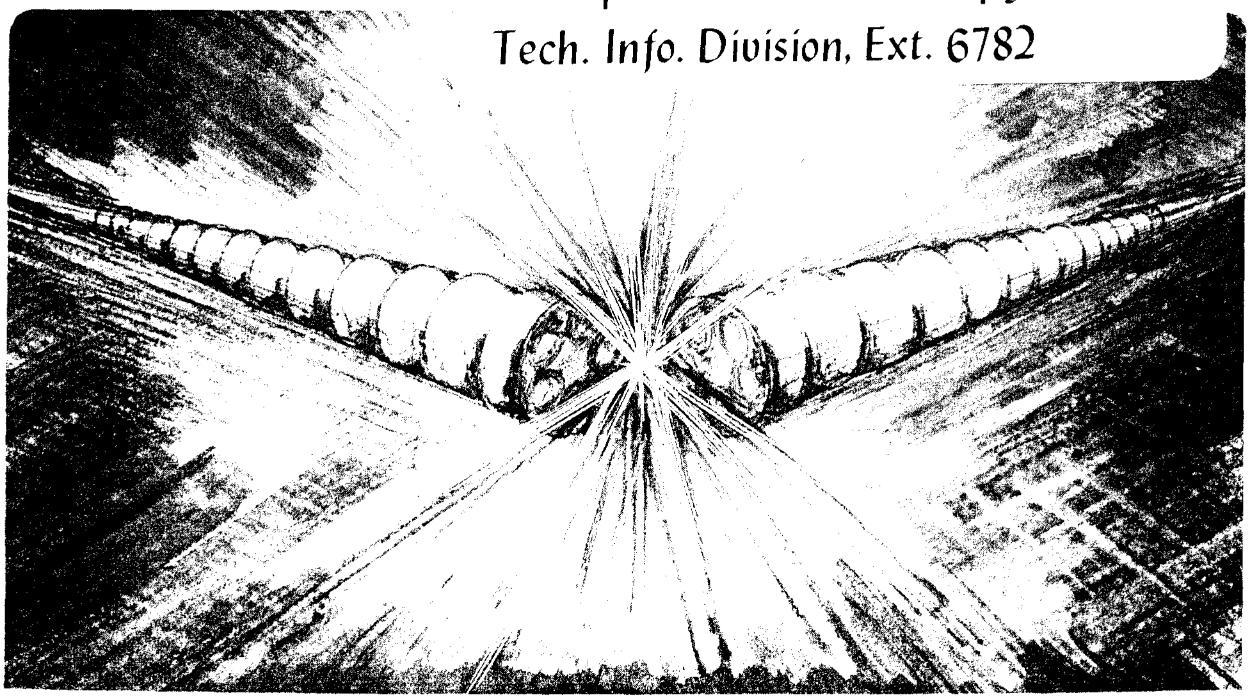

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Longitudinal Resistive Stability of an
Intense Charge Bunch in a Linear Accelerator

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ABSTRACT

A simple, but realistic, model is used to theoretically investigate the longitudinal stability of a non-relativistic bunch in the limit of small wall resistivity compared to self-reactance. It is shown that to lowest order -- and in contrast with an infinitely long beam -- that an intense bunch is stable against longitudinal collective modes. It is concluded that an induction linac remains a viable option as a driver for heavy ion inertial fusion.

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Heavy ion inertial fusion is envisioned as having either an rf linac or an induction linac as a driver. The rf linac has its major current multiplication in storage rings and a short induction section at top energy, so as to reach the requisite power level. The manipulation of beams of particles in and out of the storage rings, and their behavior while in the storage rings poses problems which must be overcome for this approach to prove practical. (See Reference 1, and references cited therein.)

The induction linac, on the other hand, is envisioned as accelerating significant current (many hundreds of amperes) for lengths requisite to attain 10 GeV (many kilometers). In this approach stability of an intense bunch of particles is essential; much effort has been devoted to this subject.[2,1] The requirement of transverse stability puts restrictions on the linac which appears tolerable, but a significant question has surrounded the longitudinal motion.

For a very long bunch it is easy to take the analysis which has been presented for circular machines and apply it to a linear, longitudinally uniform structure.[3] Firstly, one notes that one is "below transition" or in a positive mass regime so that only in the presence of resistivity is there instability. One finds, for above threshold, that the e-folding length, λ, is given by:

$$\lambda^{-1} = (R'/Z_o) \left[\frac{4\pi^2 q^2}{(1+2\ \ell n\ (b/a))} \frac{M_p}{M} \cdot \frac{N}{L} \cdot r_p \right]^{1/2} \qquad (1)$$

where,

$Z' \equiv R' + iX'$ = the impedence per unit length

N/L = line density of ions

r_p = classical proton radius

Z_o = free-space impedence (or 377 ohms)

q = degree of ionization of the ions

M_p/M = mass of the ions in units of the proton mass

Putting in R' = 200 ohms/meter, $q = 2$, $M_p/M = 1/200$, $N/L = 10^{15}/20$ meters, $b/a = 1.5$. Eq. (1) yields a length, λ, of 300 meters which is uncomfortably short for a linac of the length required. Of course, one should not use the theory for long bunches. Nevertheless, the calculation just presented gives one pause about the use of an induction linac for heavy ion fusion.

On the other hand, Kwang Je Kim has given a calculation for a finite bunch of <u>uniform</u> charge and with a step-function distribution in momentum.[4] Here the modes are exactly the same as for an infinitely long beam; namely, for any wavenumber there is a growing wave going backwards (in the beam frame) and a damping wave going forward. The bunch provides boundary conditions which exactly match these waves together so that there is no net growth.

Naturally a real bunch does not have uniform density up to sharp ends, and hence the optimistic results of Kim may be considerably modified. Just how much has been the subject of theoretical work and numerical simulation studies.[5] Unfortunately, neither the analytic or numerical studies have yet answered the question as to what growth to expect in a realistic bunch.

In the work described here we prove that in lowest order in a variety of things, but for <u>any</u> finite bunch there is no net growth of a resistive instability. In short, the result of Kim is very general. We shall detail the meaning of higher order, but clearly it implies a considerable increase of λ. We conclude that a practical linear induction accelerator for fusion will not subject intense particle bunches to significant longitudinal instability and, hence, from this very important theoretical point of view the driver remains a viable and interesting possibility.

The ions, which are collisionless, are described by the non-relativistic (it is possible to remove this restriction) non-linear Vlasov equation

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z} + \frac{qe}{M} E \frac{\partial}{\partial v}\right) f(z, v, t) = 0 , \tag{2}$$

where z is the longitudinal coordinate, v is the velocity associated with the z coordinate, e is the proton charge, t is the time and the ion distribution function is the unknown f. The longitudinal electric field consists of an applied field, E_A, and a functional $E_s\{n\}$ of the line charge density n(z,t) where

$$n(z, t) = \int f(z, v, t)dv , \tag{3}$$

We shall obtain an integral equation for the perturbed charge density and, finally after a number of assumptions, a differential equation for the Laplace transform (in time) of the perturbed charge density.

First we see that a stationary distribution $f_o(z, v)$ satisfies

$$\left(v\frac{\partial}{\partial z} + \frac{qe}{M}\left(E_A + E_s\left\{n_o\right\}\right)\frac{\partial}{\partial v}\right) f_o\ (z, v) = 0 \tag{4}$$

where n_o is the charge density associated with f_o. We linearize the Vlasov equation, Eq. (2), and obtain for the perturbed distribution function f_1, and its associated charge density n,:

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial z} + \frac{qe}{M}\left(E_A + E_s\left\{n_o\right\}\right)\frac{\partial f_1}{\partial v} = -\frac{qe}{M} E_s\left\{n_1\right\}\frac{\partial f_o}{\partial v} \tag{5}$$

We can formally integrate Eq. (3) over unperturbed orbits giving

$$f_1(z, v, t) = J - \frac{qe}{M}\int_0^t dt'\iint dz'dv'\ G(z', v', t', z, v, t)\ E_s\left\{n_1\ (z'', t'')\right\}\frac{\partial}{\partial v} f_o\ (z', v') , \tag{6}$$

where E_s is evaluated at z' and t', J is the initial value term and G is

$$G(z', v', t', z, v, t) = \delta\left(z - z^{orbit}(t')\right)\delta\left(v - v^{orbit}(t')\right) . \tag{7}$$

The unperturbed orbits are described by z^{orbit} and satisfy the boundary conditions that z^{orbit} is z' at t' = t. Laplace transforming in time and then integrating over velocity we obtain an equation for the eigenmodes once we drop the initial value term. Thus we obtain

$$\tilde{n}_1(z, s) = -\frac{qe}{M}\int dz'dv'dv\ \tilde{G}(z', v', z, v, s)\ \tilde{E}_s\left\{\tilde{n}_1(z'', s'')\right\}\frac{\partial f_0(v', z')}{\partial v'} \tag{8}$$

where we have indicated Laplace transform quantities with a tilde and we have used the fact that G and E_s are functions only of time differences. Eq. (8) is the desired integral equation for the perturbed line charge density.

We now assume that the ions are unaffected by the unperturbed E field, which is true to first approximation, but not generally. This is equivalent to ignoring synchrotron motion in f_0 and is a valid approximation in practice. That is, we take

$$G(z', v', t'; z, v, t) = \delta(v - v')\ \delta(z - z' - v'(t - t')) \tag{9}$$

We also ignore thermal velocities in the equilibrium distribution; i.e. we take

$$f_0(z, v) = n_0(z)\ \delta(v) \tag{10}$$

In addition we specify the form of E. We take, with $E = E_A + E_s$, firstly,

$$\frac{qe}{M} E_A = -\nu_0^{\ 2} z + R'c , \tag{11}$$

where ν_0 is the synchrotron oscillation frequency in the absence of space charge and the constant c is related to a phase shift in the applied voltage taken so that in the presence of resistivity the bunch moves at constant speed. Secondly, we take[3]

$$E_s = -q\ e\ g\ \frac{\partial n}{\partial z} - R'\ V_B\ n\ q\ e \tag{12}$$

where the geometrical factors, g, is approximately, given a beam of radius a in a conducting tube of radius b:

$$g = (1 + 2\,\ell n\, b/a)\ . \tag{13}$$

The term in n is due to resistivity, and V_B is the bunch average speed.

This form for E_s is only valid, for complicated finite bunch modes, under special circumstances. In general our analysis must be done in the bunch frame (so as to obtain a dispersion relation), while boundary conditions relating E and I are only simple for oscillatory in time modes in the laboratory frame (Leontovich boundary conditions). The transformations back and forth bring corrections to Eq. (12).

With these forms for E, and the assumptions of Eqs. (9) and (10), Eq. (8) takes the form:

$$\frac{(qe)^2}{M}\, g\, n_o(z)\, \frac{\partial^2 \tilde{n}_1(z,\, s)}{\partial z^2} + \frac{(qe)^2}{M}\left(g\, \frac{dn_o(z)}{dz} + R'\, V_B\, n_o(z)\right)\frac{\partial \tilde{n}_1(z,\, s)}{\partial z}$$
$$- \left(s^2 - \frac{(qe)^2}{M} R'\, V_b\, \frac{dn_o(z)}{dz}\right)\tilde{n}_1(z,\, s) = 0 \tag{14}$$

Eq. (14) is our desired differential equation for the Laplace transformed perturbed charge density $\tilde{n}_1\ (z,\, s)$.[6] For a beam of uniform charge we can immediately solve Eq. (14) and find that

$$\tilde{n}_1\ (z,\, s) \sim e^{ikz}\ , \tag{15}$$

with a quadratic to be solved for s. The real part of s is, for $(R'/X') << 1$,

$$\mathrm{Re}\ s = \frac{R'V_B}{2}\,(qe)\sqrt{\frac{n_o}{Mg}} \tag{16}$$

in agreement with Eq. (1) when it is recalled that the impedance per unit length is defined through $E = -Z'I$ so that for $\tilde{n}_1$ given by Eq. (15) the reactance per unit length, X', is given by $X' = kg/V_B$.

We are now in a position to do a perturbation analysis for $(R'/X') << 1$ on Eq. (14) and Eq. (4). Expanding these two equations leads to a form for

Eq. (14) which can be written as

$$M_0\, \tilde{n}_1(z, s) + R'\, M_1\, \tilde{n}_1\, (z, s) = 0 \tag{17}$$

where the operators M_0 and M_1 are:

$$M_0 = \frac{(qe)^2}{M}\, g\, n_0(z)\, \frac{\partial^2}{\partial z^2} + \frac{(ge)^2}{M}\, g\, \frac{dn_0(z)}{dz}\, \frac{\partial}{\partial z} - s^2 \tag{18}$$

$$M_1 = \frac{(qe)^2}{M}\, g\, n_{01}(z)\, \frac{\partial^2}{\partial z^2} + \frac{(qe)^2}{M}\, g \left(\frac{dn_{01}(z)}{dz}\right)\frac{\partial}{\partial z} + \frac{(qe)^2 V_B}{M}\left(n_0(z)\, \frac{\partial}{\partial z} + \frac{dn_0}{dz}\right) \tag{19}$$

and n_{01} is the correction in the equilibrium $n_0(z)$, given by Eq. (4), due to R'.

Since M_0 is of self-adjoint form we obtain from perturbation theory that

$$\delta s_n = \frac{R' \int \tilde{n}_n\, M_1\, \tilde{n}_n}{\int \tilde{n}_n\, \frac{\partial M_0}{\partial s}\, \tilde{n}_n} \tag{20}$$

where $\tilde{n}_n$ is the $n\underline{\text{th}}$ mode of

$$M_0\, \tilde{n}_n = 0 \tag{21}$$

It is easy to see that from Eq. (20) we re-derive Eq. (16), and hence Eq. (1), with a bunch that is infinite in extent. We can show, employing perturbation theory, that if n_0 is even then n_{01} is odd and hence that M_1 is odd. For non-degenerate eigenvalues s_n and $n_0(z)$ zero outside, but not inside, a finite domain it is clear that n_n is of definite parity. It follows, from Eq. (20), that δs_n is zero.

This completes our proof. It is also possible -- and this will be described elsewhere (along with details of the work reported here) -- to explore corrections to the theorem. We have computed them explicitly and find as expected that the growth distance is greatly increased over that given by Eq. (1).

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[3]V.K. Neil and A.M. Sessler, Rev. Sci. Insti. 36, 429 (1965).

[4]Kwang Je Kim, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980).

[5]A. Sternlieb, private communication.

[6]Note that Eq. (14) follows from Eq. (9) by taking moments, making the corresponding approximations to Eqs. (9), (11) and (12), and closing the heirachy by neglecting the thermal spread in f_1. However the method in the paper is superior for it allows us to study corrections to Eq. (14). Note, also, that Eq. (14) can be employed with any $n_o(z)$ provided that E_A is consistent with $n_o(z)$.

[7]P. Morse and H. Feshbach, Methods in Theoretical Physics, Part II (McGraw-Hill, New York, 1953), p. 1015.